Time Derivative Isosurface Gradient

Grisafi Finance Rheology Learning Center
Steven J. Grisafi, PhD

A toy market composed of m component assets has a zero contour level isosurface for its money potential that is cast within the m dimensions of the market space. Any surface cast within the m dimensions of the toy market space would possess $m-1$ dimensions. If the zero contour level isosurfaces for m consecutive time periods of an m dimensional toy market are cast simultaneously with the market space they would intersect at a single point consisting of m elements. For a three component toy market the intersection of three consecutive time period isosurfaces is an ordinary point. As time passes, a rolling consecutive time period intersection point traces a smooth curve within the toy market space. Time differentiation of the trajectory of the intersection point can be found numerically through the use of finite differences. Each component of the time derivative of the intersection point presents an average measure of the time rate of change for each asset component of the toy market. Through interchange of the order of differentiation measures of the time rate of change for the components of the gradient of the zero contour level isosurface can also be found.

Let $S(x, y, z)$ designate a surface within the space of a three asset toy market. Adopting the use of a quadratic isosurface yields the form

$$S(x, y, z) = ax^2 + bx + cy^2 + dy + ez^2 + fz + g \tag{1}$$

The undetermined coefficients a, b, c, d, e, f, g are to be evaluated however one chooses to do so. The quadratic isosurface $S(x, y, z)$ is to be set equal to zero when it is designated as the zero contour level isosurface for evaluation of the money potential. The independent variables x, y, z can represent any desired measure of price or income. The total differential for an increment of the isosurface is

$$dS = \frac{\partial S}{\partial x}dx + \frac{\partial S}{\partial y}dy + \frac{\partial S}{\partial z}dz \tag{2}$$

The partial derivatives are all linear functions of the independent coordinates and readily evaluated for all points within the market space. Each differential of the independent coordinates can be divided by its others forming the components of the isosurface gradient. The gradient components are

$$G_{zx} = \frac{dz}{dx} \tag{3}$$

$$G_{yx} = \frac{dy}{dx} \tag{4}$$

$$G_{zy} = \frac{dz}{dy} \tag{5}$$

Note that symmetry dictates

$$G_{yx} = (G_{xy})^{-1} \tag{6}$$

When using the definitions from eqs. (3) through (6) within eq. (2), two coupled equations can be formed

$$G_{zx}\frac{\partial S}{\partial z} + G_{yx}\frac{\partial S}{\partial y} = -\frac{\partial S}{\partial x} \tag{7}$$

$$G_{zy}\frac{\partial S}{\partial z} + G_{xy}\frac{\partial S}{\partial x} = -\frac{\partial S}{\partial y} \tag{8}$$

The two equations are not independent. They can be made to reduce to the single constraint

$$1 + \frac{2ez+f}{2cy+d}G_{zx} = \big(1 + G_{zx}\frac{2ez+f}{2ax+b}\big)^{-1} \tag{9}$$

So one sees that the two horizontal components of the gradient with respect to the vertical coordinate are linked. The purpose of the constraint is to maintain proper accounting of all money balances. Such balances can go astray when fitting undetermined coefficients to time series data.