Basecoin: Exchange Rate Equilibrium

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The Basecoin is proposed as a replacement for the € as the medium for international exchange amongst member nations of the European Monetary Union. Within each member nation of the Union, sovereign currencies are to be reinstated as the medium of exchange for all domestic commerce. Only international commerce among member nations of the European Monetary Union is to utilize the Basecoin as the currency for commerce. Should the European Monetary Union choose to adopt the Basecoin with this proposed capacity, the reinstitution of all national currencies would require their evaluation relative to the Basecoin valuation as it is pegged to the value of the soon to be relinquished €. For this purpose the following method is proposed utilizing a money potential to assess the fundamental valuation for each sovereign currency.

One seeks to solve the money potential for its gradient, instead of merely using the gradient of a particular isosurface, to be able to identify the existence of wave propagation, the diffusivity, and other material properties of a market system. The isosurface is a single instance of a data set and not representative of the fundamental behavior of the system. It should be viewed as no more than a particular solution from which we would seek the general solution. Our method is to build a money potential upon an isosurface formed from the exchange rates of two selected currencies relative to a third. Additional currencies may be included parametrically within the isosurface, but do not serve as independent coordinates for the money potential. By taking the ratio of the exchange rates of the two selected currencies with respect to the third currency one eliminates the third while forming the best estimate of the exchange rate of the two selected currencies to one another. The parametric inclusion of additional currency exchange rates, with respect to the selected third currency, incorporated within the isosurface is intended as as sort of mean field effect of other currency exchange rates upon the exchange rate of the two particular currencies of interest.

Let S designate a surface within the space of an $m+2$ currency market. A linear isosurface is a poor choice upon which to build a money potential. However, a linear isosurface permits an algebraic solution, which for the purpose of this instruction, is more suitable than choosing to use a more accurate isosurface. The concomitant numerical computations required for building a money potential upon a quadratic isosurface obscure the clarity one achieves through the use of analytic solutions. For estimations of importance one should always use a quadratic isosurface, or better, upon which to build the money potential. Numerical computations would not permit an explicit, concise, description of the methodology. Hence, for this purpose of illustration, consider a linear isosurface of the form

$$S(x, y) = ax + by + c \tag{1}$$

The variables x and y denote the exchange rate for two pertinent currencies within the monetary union relative to some arbitrary currency external to the monetary union. The undetermined coefficients a, b, c are to be evaluated however one chooses to do so. However, the zero order constant, c, may be taken as a linear composition of other m additional sovereign currencies within the union. Thus

$$c = \sum_{j=1}^{m} c_j z_j \tag{2}$$

The isosurface $S(x, y)$ is to be set equal to zero when it is designated as the zero contour level isosurface for evaluation of the money potential. Let Ψ denote the money potential. For this two dimensional system its governing equation is

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = S(x, y) \tag{3}$$

A general solution can be readily found for Ψ when the independent variables of equation (3) are transformed to polar coordinates. Let ζ denote the magnitude of the polar coordinates' phasor and θ its phase angle then equation (3) becomes

$$\frac{1}{\zeta}\frac{d}{d\zeta}(\zeta\frac{d\Psi}{d\zeta}) = [a\sin\theta + b\cos\theta]\zeta + c \tag{4}$$

As a consequence of its symmetry, the diffusion equation is no longer a partial differential equation, but is now an ordinary differential equation. It is readily integrated twice to yield the solution

$$\Psi = \frac{\zeta^3}{9}[a\sin\theta + b\cos\theta] + c\frac{\zeta^2}{4} + d\ln(\zeta) + e \tag{5}$$

where d and e are constants of the integrations. Polar money coordinates are of little use to us, so we transform back to the Cartesian money coordinates:

$$\Psi = (x^2 + y^2)\Big[\frac{ax + by}{9} + \frac{c}{4}\Big] + d\ln(\sqrt{x^2 + y^2}) + e \tag{6}$$

Since currency can be worthless, the point $(x = 0, y = 0)$ is admissible. Therefore, one must set the constant $d = 0$. To obtain the values of (x, y) at locations within the money potential where they occur at either a local maximum or minimum one differentiates equation (6) with respect to both Cartesian money coordinates to obtain two equations for two unknowns. Setting the first derivative equations to zero places the money coordinates at locations within the money potential where its gradient vanishes. Regardless of whether such points are maxima, minima, or saddle points, they are locations where there is no driving potential to change their value. As such, one may consider such points as equilibria even if only meta-stable. Differentiating equation (6) with respect to both of its independent variables and setting the resulting equations equal to zero yields

$$a\frac{x^2}{3} + (\frac{2by}{9} + \frac{c}{2})x + \frac{ay^2}{9} = 0 \tag{7}$$

$$b\frac{y^2}{3} + (\frac{2ax}{9} + \frac{c}{2})y + \frac{bx^2}{9} = 0 \tag{8}$$

Equations (7) and (8) are simultaneous quadratic equations for the two coordinate variables x and y. The two simultaneous equations always have four roots. Since the trivial solution is admissible one of the roots is always (0,0). The equations are real so complex roots must enter only in pairs. The equations can be solved algebraically using the quadratic formula to yield

$$x = \left[-(\frac{2by}{9} + \frac{c}{2}) \pm \sqrt{(\frac{2by}{9} + \frac{c}{2})^2 - \frac{4}{27}y^2}\right] / (\frac{2a}{3}) \tag{9}$$

$$y = \left[-(\frac{2ax}{9} + \frac{c}{2}) \pm \sqrt{(\frac{2ax}{9} + \frac{c}{2})^2 - \frac{4}{27}x^2}\right] / (\frac{2b}{3}) \tag{10}$$

The quadratic formulae of equations (9) and (10) indicate that there are only the two possibilities: There are either four real roots or there are two real roots and two complex roots. The two complex roots always yield the same real value for their ratio. If there should be a real root along with the root of the trivial solution, and should that value be unsuitable, then one ought evaluate the ratio of the magnitudes of the complex roots.

As a demonstration of the procedure, consider an isosurface formed of the two Cartesian money coordinates representing the exchange rates for the Euro dollar, €, and the Pound Sterling, £, each taken per unit Bitcoin, $\mathbb{B}$. One seeks to define the exchange rate of Sterling per unit Euro from time series data taken of the Euro per Bitcoin and Sterling per Bitcoin exchange rates. Although one could easily find the exchange rate of Sterling per Euro directly, the procedure is meant to devise a method for defining exchange rates between currencies not previously having any such exchange Thus, the Bitcoin serves as the third arbitrary currency, to which the pertinent currencies of interest are measured relative. As an an example of a confounding influence upon the measurement of the Sterling per Euro exchange rate let us include parametrically within the isosurface measurements of the USA dollar, \$, and the Basecoin, $\mathfrak{B}$, both of whose values are also measured per unit $\mathbb{B}$. Since our intention is to replace the € as some equivalent measure of $\mathfrak{B}$, let us form the isosurface as a linear combination of €, £, and \$ such that

$$\mathfrak{B} = a\text{€} + b\pounds + c_0\$ \tag{11}$$

where all are measured per unit $\mathbb{B}$. To evaluate a money potential, our isosurface must be the zero contour level isosurface, therefore equation (1) takes the form

$$S(\text{€}, \pounds) = a\text{€} + b\pounds + c_0\$ - \mathfrak{B} \tag{12}$$

This then indicates that the zero order constant c of equation (2) becomes

$$c = c_0\overline{\$} - \overline{\mathfrak{B}} \tag{13}$$

Here the overbar indicates that an average value of the variable taken over the time series data set should be used. To exclude the effects of parametrically introduced currencies, one would evaluate the constants a, b, c for an isosurface defined solely by (€, £).

One now associates the variable € with the Cartesian money coordinate x, and £ with y, in equations (9) and (10). Numerical values for the constants a, b and c are to be determined by fitting the surface defined by (11) to time series data for the per unit $\mathbb{B}$ exchange rates of €, £, \$ and $\mathfrak{B}$. Then upon substitution of those numerical values for a, b and c into equations (9) and (10), numerical values for the zero gradient locations of the money coordinates (€, £) can be found.

For a time series data set consisting of twenty-one daily recordings, the linear isosurface appears to be adequate for building a money potential. But only when the confounding effects from the parametrically introduced currencies are excluded. A straightforward ratio of the average exchange rates for the € and the £, both taken per unit $\mathbb{B}$, yields an estimate of the € equaling 73% of a £. Using the same time series data set for the exchange rates, but excluding the effect of the \$ and the $\mathfrak{B}$ when defining the zero contour level linear isosurface for the money potential, yields an estimate of the € equaling only 71% of a £. If the confounding effects of the \$ and the $\mathfrak{B}$ exchange rates, both taken per unit $\mathbb{B}$, are included parametrically within the determination of the zero contour level linear isosurface, then the money potential predicts equivalence for the values of the € and the £. One may consider the money potential estimate, excluding the parametric confounding exchange rate effects, to suggest that the 2% discrepancy from the straightforward average ratio indicates that the £ is slightly overpriced. However, the money potential prediction, which includes the confounding parametric exchange rate effects, and asserts equivalence for the value of the € with the £, may just be a failure of the linear isosurface to conform accurately to the time series data.

One may now apply this analysis to establish exchange rates for $\mathfrak{B}$ and the to be reinstated sovereign currencies for the member nations of the European Monetary Union. Defining a linear isosurface composed solely by the locus of exchange rate points for $(\mathfrak{B}, €)$ predicts an equilibrium exchange rate of approximately four $\mathfrak{B}$ for every three €. Hence, for member nations of the European Monetary Union with a strong national economy, such as Germany, the German Finance Minister would seek to reinstate the sovereign currency at a rate of approximately 4/3 $\mathfrak{B}$ per Deutsch Mark. Member nations with a less strong economy, such as Greece, would seek a lesser exchange rate, perhaps, more on the order of one Greek Drachma per $\mathfrak{B}$. All nations would receive one $\mathfrak{B}$ for each € they possess.