Stream Space Eigenvalues
BTC vs. BCH Exchange Market
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In anticipation of the forthcoming analysis of the exchange rates between Bitcoin (BTC) and Bitcoin Cash (BCH), soon to be posted here at the Grisafi Finance Rheology Learning Center, an exposition of the development for the analysis regarding the exchange stream space is in order. Unlike the capacitance phase space, $\{\eta, \mu\}$, the inductance phase space, $\{\xi, \zeta\}$, presents time averaged values of the system trajectory that have actually been achieved. The capacitance phase space presents the trajectory of optimal values the system would be required to reach if price diffusion equilibrium were to be achieved. While it is expected that, absent of all other externalities, the activity of market participants would drive the two trajectories to join one another, the two may not necessarily converge.

In the forthcoming analysis, to be posted under the Bitcoin Cash abbreviation BCH, market dynamics of the exchange rates for the ratio of the Bitcoin price relative to the Bitcoin Cash price, BTC/BCH, will be presented. As is customary in the application of the methods of finance rheology, the analysis begins with the development of the zero level contour isosurface for the market system. When using an isosurface to build upon it a price potential, a quadratic isosurface is preferred. When evaluating currency exchange rates, a linear isosurface usually suffices. In the forthcoming analysis, both a quadratic isosurface will be used to build a price potential, and a linear isosurface is to be used to evaluate the exchange stream space. The primary discretion difficulty of an exchange stream analysis is deciding upon which alternative trades are most important to the market under consideration. For the Bitcoin versus Bitcoin Cash market, the most pertinent third party currency trades are anticipated to be with respect to trades denominated in the US Dollar, \$, the EURO Dollar, €, and the Pound Sterling, £. Hence, we construct the linear zero level contour isosurface as

$$0 = a\$ + b€ + c£ \tag{1}$$

Here, each currency symbol is used to indicate the value of the price ratio BTC/BCH measured with respect to each particular sovereign currency. The parameters a, b and c are undetermined coefficients to be evaluated from time series data using some ordinary least squares technique.

As shown previously in the FINRHEO research article "Stream Space Inductance: Currency Exchange Via Universal Stream Inductance", in order to evaluate a closed form analytical solution for the optimal roots defining the capacitance $\{\eta, \mu\}$, one would need to rearrange the isosurface equation (1) such that the third parameter c stands alone as a constant term. A slightly different approach will be taken here since we will not seek to define the capacitance phase space directly. Instead we will seek the eigenvalues of the inductance phase space. Such quantities do not necessarily indicate optimal values for the market system; but, rather, indicate characteristic behavior of the market system that it actually achieves. Since experience indicates that markets rarely achieve equilibrium it may very well be preferable to guide one's behavior by what the market actually does achieve than by what it should achieve if market participants were truly rational. Yet, the analysis is not very different from the construction of the capacitance space. All that is different is that we do not seek an optimal solution.

All three terms of equation (1) are linear. Consequently, all three coefficients a, b and c are of equal stature. While each may indicate a different currency, none is special from the others. We begin as though we were to apply a row reduction scheme to a linear system of equations. As was shown previously in the stream space inductance scheme, we could divide two of the parameters by the third such that we would then bring the third term to the left side of the linear equation. But since all parameters are of equal stature we should do this for all of them. The result of doing so is to construct a linear system of three simultaneous equations

$$0 = \frac{a}{c}\$ + \frac{b}{c}€ + £ \tag{2}$$

$$0 = \frac{a}{b}\$ + € + \frac{c}{b}£ \tag{3}$$

$$0 = \$ + \frac{b}{a}€ + \frac{c}{a}£ \tag{4}$$

This linear system of equations is singular because there is actually only one equation with three unknowns. To evaluate any solutions to this linear system of equations we first cast it in matrix form as

$$\mathbf{0} = \mathbf{Mx} \tag{5}$$

Where $\mathbf{0}$ is a 3 by 1 column vector of zeros and

$$\mathbf{M} = \begin{bmatrix} 1 & \frac{b}{a} & \frac{c}{a} \\ \frac{a}{b} & 1 & \frac{c}{b} \\ \frac{a}{c} & \frac{b}{c} & 1 \end{bmatrix} \tag{6}$$

and

$$x = \begin{bmatrix} \$ \\ € \\ £ \end{bmatrix} \tag{7}$$

For any solution to this system of linear equations to exist the determinant formed with the eigenvalues of the system must vanish

$$\det \begin{bmatrix} 1-\lambda & \frac{b}{a} & \frac{c}{a} \\ \frac{a}{b} & 1-\lambda & \frac{c}{b} \\ \frac{a}{c} & \frac{b}{c} & 1-\lambda \end{bmatrix} = 0 \tag{8}$$

The characteristic polynomial formed by evaluation of the determinant in equation (8) always has two solutions. One solution is trivial with one eigenvalue equal to unity and a second equal to two. There is no third eigenvalue. The second solution always casts one eigenvalue equal to zero with the other eigenvalue equal to three. The eigenvectors associated with each of the two nonzero eigenvalues are orthogonal to one another. We need to include the contribution of both eigenvalues simultaneously. In order to do that we must construct the quadratic form for the matrix $\mathbf{M}$. To create the quadratic form we notice that we can multiply both sides of equation (6) from the left with the transpose $\mathbf{M^T}$ of the matrix $\mathbf{M}$ and the equation remains unchanged:

$$\mathbf{M^T 0} = \mathbf{M^T M x} \tag{9}$$

Thus, we seek the eigenvalues of the quadratic form $\mathbf{M^T M}$. Just as before, there are always two solution sets for the eigenvalues. We disregard the first solution, which is identical to the solution for $\mathbf{M}$. The second solution yields one eigenvalue as zero with the other as:

$$m = \frac{(b^2 + a^2)c^4 + (b^4 + 3a^2b^2 + a^4)c^2 + a^2b^4 + a^4b^2}{a^2b^2c^2} \tag{10}$$

We designate the nonzero eigenvalue as m. The eigenvectors associated with these two eigenvalues are also always orthogonal to one another. Yet, both contain complete information for the original matrix $\mathbf{M}$. The eigenvalue m serves as our estimate tracking the time series.