Conic Yields

Empirical Yield Transmission

Grisafi Finance Rheology Learning Center

Steven J. Grisafi, PhD

The primary purpose for studying the dynamics of the yield curve for United States Treasury debentures is to identify a mechanism by which the central bank can alter the curvature of the yield curve through the manipulation of overnight borrowing and lending transactions that the central bank undertakes with financial institutions having accounts with the bank. While the central bank can participate in overnight transactions with various financial institutions it remains to be seen how interest rates set during those transactions can propagate an effect onto transactions having expiration time periods far exceeding the overnight term. It appears plausible that market participants determine the characteristics descriptive of a market through the conditions that they set with one another during the course of their transactions. Yet it appears less plausible to assume residual effects lingering beyond the expiration terms of those transactions causing effects beyond that particular market. This begs the question: How does the central bank expect the characteristics of its overnight transactions to influence the characteristics of transactions yet to come elsewhere within a national economy?

The impetus for the present analysis is the observation found within figure 1 that certain derivative quantities of the money potential, Ψ, exhibit marked stability with a clearly defined structure as the time evolution of the dynamics progresses. Figure 1 presents as its ordinate the quantity I refer to as the diffusivity ratio. Strictly speaking the ratio of the integral of the money potential, Ξ, over the integral of the zero contour level isosurface, Γ is not the diffusivity. To evaluate the diffusivity one must use differential quantities. However, the ratio Ξ/Γ provides a quick and simple means of gauging any relationship the diffusivity may enter into. The abscissa of the graph within figure 1 is the ratio of the two horizontal components for the gradient of the money potential, $GradX$ and $GradY$. The horizontal components are generally referred to as the X and Y components. In our particular case under consideration the X component would be the yield of the three month treasury bill and the Y component would be the yield of the five year treasury note. To avoid the use of the ubiquitous X and Y within our nomenclature for a particular case let us designate the yield of the three month treasury bill as μ and the yield of the five year treasury note as η.

The graph of Ξ/Γ versus $GradY/GradX$ exhibits a shape similar to a portion of a conic section, most notably an ellipse. The shape need not be an actual ellipse since all we require is its stability through time evolution. Yet this similarity to an ellipse suggests the following function for an empirical curve-fit to the data points:

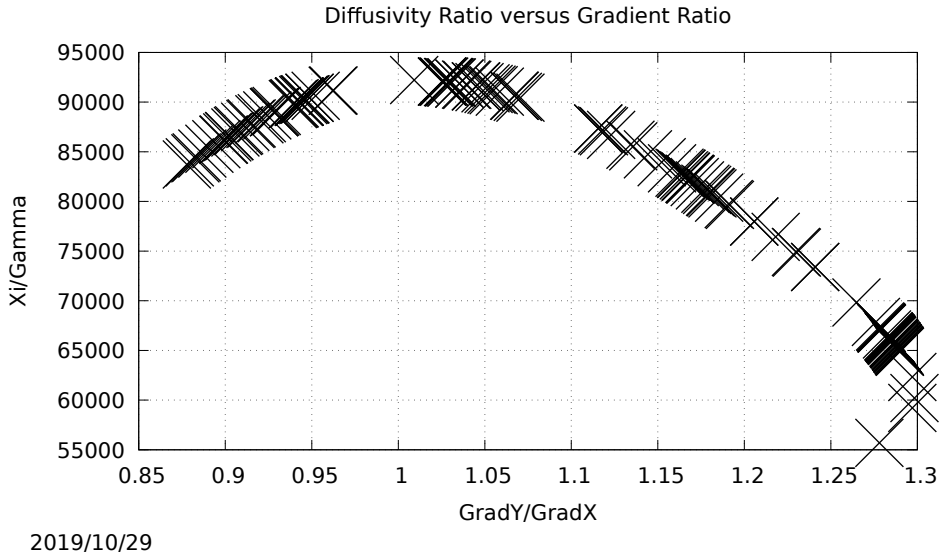

Figure 1: U.S. Treasury Yield Curve

$$(\xi - h)^2 + n(\zeta - k)^2 = j^2 \tag{1}$$

Within equation (1) the dynamic variables ξ and ζ represent the abscissa and ordinate of our figure 1, $GradY/GradX$ and Ξ/Γ, respectively. The parameters h, k, j are undetermined coefficients to be evaluated using any ordinary least squares regression. With no loss of generality, the fourth parameter, n, may be set to unity since we are unconcerned with the positioning of the center, foci, and directrices of the conic section we seek to describe. Doing so implies that the parameters h, k, j would not have their customary associations with those characteristic quantities descriptive of conic sections. As a consequence of using ratios within the ordinate and abscissa of figure 1, there will be only one degree of freedom for the relationship that we seek. There is only one parameter of interest, which is to be determined from the values found for h, k, j, in the differential relationship we establish.

The ongoing study of the Treasury Yields conducted at the Finance Rheology Learning Center indicates a non-zero relaxation time ranging between thirty and forty days. This magnitude suggests that any perturbations to the market caused by central bank interventions ought to have a lingering effect reaching only the three month treasury bill. The current analysis reported here seeks to demonstrate what effect, if any, can be expected to propagate to longer treasury yield maturities as a result of market participants responding to the actions of the central bank.

The abscissa and ordinate of the graph in figure 1 are differential and integral operations upon the money potential Ψ for the interest rate yield curve. Substituting the differential operations into equation (1) shows

$$\big((\frac{\partial\Psi}{\partial\eta})/(\frac{\partial\Psi}{\partial\mu}) - h\big)^2 + (\frac{\Xi}{\Gamma} - k)^2 = j^2 \tag{2}$$

Equation (2) can be simplified immediately to show

$$\big(\frac{\partial\mu}{\partial\eta} - h\big)^2 + (\frac{\Xi}{\Gamma} - k)^2 = j^2 \tag{3}$$

The integrals Ξ and Γ are complicated expressions, depending not only upon time, but also upon the dynamic variables, μ and η. If they were constants, the first order differential equation shown as equation (3) could be immediately solved. Both Ξ and Γ are slowly varying functions of time and their ratio is even more so. Yet, since we are concerned with only small perturbations to the yield curve, a differential relationship is all we seek or require. Inspection of equation (3) suggests that we define the following simplifying notation:

$$K = h + \sqrt{j^2 - (\frac{\Xi}{\Gamma} - k)^2} \tag{4}$$

Utilizing the expression for K within equation (3) enables it to be reduced to the simple differential form:

$$d\mu = K d\eta \tag{5}$$

At the time of the writing of this article, the empirical data from the Treasury Yields web-page of the Finance Rheology Learning Center indicates that K is positive and has a magnitude on the order of 10^3. Values for the proportionality constant, K, and the coefficients h, k, j are posted daily six days a week on the Treasury Yields web-page of the Finance Rheology Learning Center. The value of K indicates that the usual central bank intervention of raising or lowering its overnight lending or deposit interest rates by 25 basis points could at most only have the effect of raising or lowering the yield on the five year treasury note by 0.025 basis points. Such a small intervention is likely to be lost in the round-off errors occurring during sales of the five year treasury notes.