Exchange Capacitance

Optimal Root Phase Space

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The exchange stream phase space of the exchange inductance, proffered earlier in the article "Stream Space Inductance: Currency Exchange via Universal Stream Inductance", presents the locus of points a currency exchange system has actually achieved, and could possibly achieve, should the system remain unaltered. There is no reason to suppose that this trajectory either displays, or converges, to an equilibrium for the currency exchange. The system equations for estimating the optimum points of a currency exchange system, proffered earlier in the article "Basecoin: Exchange Rate Equilibrium", are difficult to solve. For reference, they are presented again here below:

$$x = \left[-(\frac{2by}{9} + \frac{c}{2}) \pm \sqrt{(\frac{2by}{9} + \frac{c}{2})^2 - \frac{4}{27}y^2}\right] / (\frac{2a}{3}) \tag{1}$$

$$y = \left[-(\frac{2ax}{9} + \frac{c}{2}) \pm \sqrt{(\frac{2ax}{9} + \frac{c}{2})^2 - \frac{4}{27}x^2}\right] / (\frac{2b}{3}) \tag{2}$$

Within equations, (1) and (2), the parameters a, b, and c are determined via a regression curve-fit of an isosurface representing the zero level contour surface of the currency exchange system. The equations are two simultaneous, nonlinear, implicit equations for the roots x and y whose ratio provides an optimum point for a currency exchange transaction. This optimum point can be either a saddle point or a maximum, minimum. If it is a maximum, it is also a minimum, and *vice versa*, depending upon the direction of the trade. Such points may never actually become part of the locus of points that form the phase space for stream space inductance. These equations require a numerical method to solve for their roots. Thus, it is desirable to have a systematic procedure for defining the locus of optimal roots that these equations provide for all currency exchange systems.

To achieve our goal we define a transformation of the two roots, x and y, onto our optimal root phase space given by η and μ as:

$$\eta = \frac{ax}{c} \tag{3}$$

$$\mu = \frac{by}{c} \tag{4}$$

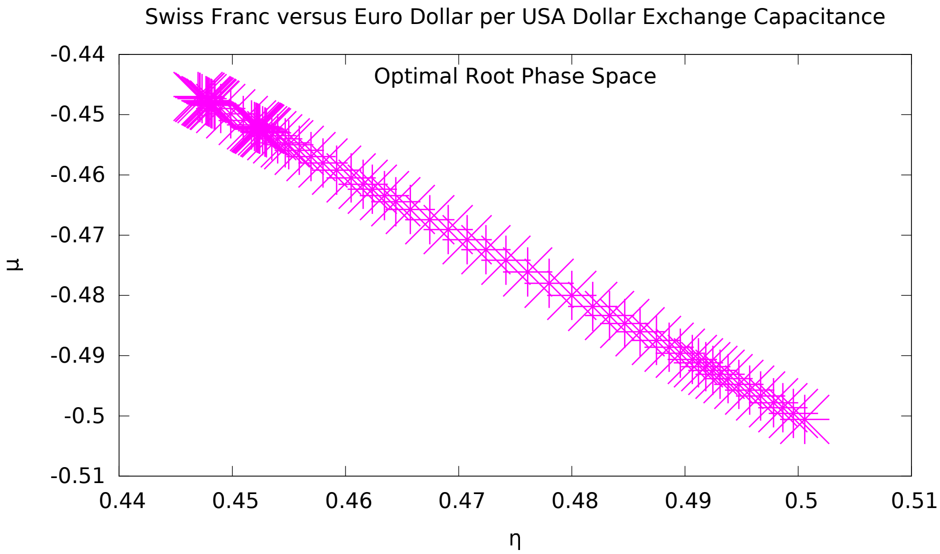

Figure 1: Swiss Franc versus Euro Dollar

The transformation utilizes the regression coefficients a, b and c. The applicability of the method requires that the phase space plot of $\{\eta,\mu\}$ defines some known functional form that persists from one exchange system of interest to another. It is customary to transform variables such that their transformation becomes linear; but there could be other equally useful graphs. In our case, we find that the transformation, defined with equations (3) and (4), provides a strong linear relation for the currency exchange systems of the USA Dollar based exchanges, and also the Basecoin exchange basis system, as shown on the Grisafi website.

Figures 1 and 2 display the success of the transformation. Within each graph of the figures, the relationship between the optimal root phase space variables can be defined as:

$$\mu = m\eta + \mu_0 \tag{5}$$

Within the simple linear relationship of equation (5), m provides the slope of the line and μ_0 provides its intercept. When the empirical data shows a strong linear relationship the method can then proceed in the same manner as for defining the stream space inductance. But in order to do so another transformation of the variables is required first. Consider the auxiliary transformation defined using the following two equations:

$$\theta = \frac{\mu}{\mu_0} \tag{6}$$

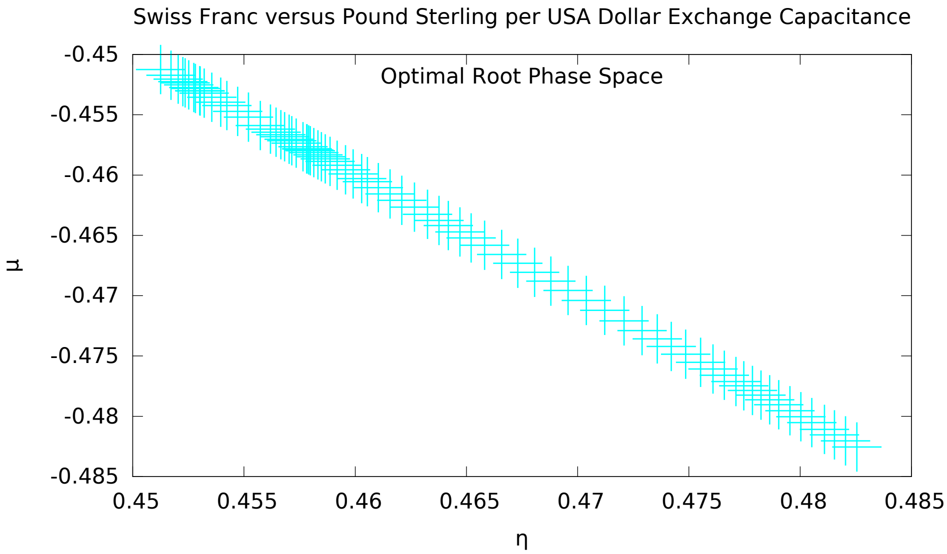

Figure 2: Swiss Franc versus Pound Sterling

$$\sigma = \frac{m\eta}{\mu_0} \tag{7}$$

Equations (6) and (7) transform equation (5) into the simple relationship:

$$\theta = \sigma + 1 \tag{8}$$

This is an algebraic equation. We can transform it into a more universal relationship by making it a differential equation. The similarity of equation (8) to equation (4) of the inductance article indicates that we can follow the same procedure used there for defining a universal exchange function. However, while one can utilize the solutions obtained previously for the stream space inductance, our further transformations define the optimal root phase space as providing a quantity distinct from the inductance function, ψ. What we are measuring now is a latent capacity of the exchange system, and not its actual performance.

We define our new quantity as the capacitance of the currency exchange system. Let us designate this new universal exchange function with the symbol χ. We call it the currency exchange capacitance. Its defining relationship to the auxiliary phase space variables is:

$$\theta = \frac{\partial\chi}{\partial\sigma} \tag{9}$$

$$\sigma = \frac{\partial \chi}{\partial \theta} \tag{10}$$

Using equations (9) and (10) within equation (8) transforms it into the following partial differential equation:

$$\frac{\partial \chi}{\partial \sigma} = \frac{\partial \chi}{\partial \theta} + 1 \tag{11}$$

Equation (11) has the same form as equation (7) of the inductance article. Therefore, its solution is of the same form:

$$\chi = \frac{\theta^2 + \sigma^2}{2} + \theta\sigma + \frac{\theta + 3\sigma}{2} + \chi_0 \tag{12}$$

In equation (12) χ_0 is a constant of the integration. It serves the same purpose as ψ_0 does for the inductance ψ. Similarly, it can be evaluated in the same manner so long as one accounts for the additional transformation to the auxiliary phase space variables $\{\sigma, \theta\}$ from the optimal root phase space variables $\{\eta, \mu\}$.